INDUSTRIAL FANNO FLOW ANALYSIS BY SIMPLE TABLE

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ABSTACT

It is interesting that ratio values of Fanno duct choke state to nozzle choke state are identical regardless of source ideal gas state and gas mole weight in case of maximum flow passing through a duct, which is called herein as "Free Flow". In case of controlled flow called herein as "Given Flow", ratio values of duct inlet pressure/temperature to duct exit choke pressure/temperature are identical regardless of source ideal gas state and gas mole weight.

And, there are certain relations if source ideal gas state and gas mole weight are different from those of nozzle in case of Free Flow and from those of a reference duct in case of Given Flow.

Industrial Fanno flow analysis can be performed straight forward for all cases easily and simply by using the universal ratio tables with the relations for different conditions, instead of using complicated try-and-error method required when performing the analysis merely by calculation.

INTRODUCTION

Industry uses Fanno equations for analysis of behavior of compressible gas in a duct. In the analysis, normally source gas state is given instead of Mach number at inlet cross section of duct, which is to be given in text book analysis of school. Further, the analysis is not be calculated straight forward, but requires iteration of calculation by try-and-error method.

In order to develop straight forward calculation method by using a table like steam table, Author tried to make a Fanno expansion table for industry, in which gas state at duct inlet and outlet cross section as well as flow rate can be found on the table with source gas state and duct friction resistance coefficient given. While making the table, Author found that there are certain relations among table values and industrial Fanno analysis for all cases can be found easily by using the relations with a simple table, which must be helpful for engineers to do the analysis quickly and easily.

NOMENCLATURE

K	: Friction resistance coefficient, K = f x Lmax / D
F	: Friction factor
L, Lmax	: Duct length(L), duct length having choke state at the exit(Lmax)
D	: Duct diameter
W	: Mass flow rate
A	: Duct cross section area
W/A, W/Amax:	: Mass flow rate per unit cross section area(W/A) Mass flow rate per unit cross section area at choke state(W/Amax) In case of nozzle, cross section area is at nozzle throat.
P0, T0, H0	: Absolute pressure(P0) and absolute temperature(T0) of source ideal gas
M1, P1, T1, H1, Vel1, SonicVel1, v1	: Mach number(M1), absolute pressure(P1), absolute temperature(T1), enthalpy(H1), velocity(Vel1), sonic velocity(SonicVel1), specific volume(v1) of ideal gas at inlet cross section of duct
P2c, T2c	: Absolute pressure(P2c) and absolute temperature(T2c) at exit cross section of duct having choke state
Pc_nozzle, Tc_nozzle	: Absolute pressure(Pc_nozzle) and absolute temperature(Tc_nozzle) at nozzle throat cross section having choke state
K	: Specific heat ratio of ideal gas, k = Cp / Cv
Cp	: Specific heat at constant pressure
Cv	: Specific heat at constant volume
G	: Acceleration of gravity, 9.80665 m/s2
J	: Mechanical equivalent of heat, 101.9716213 kg-m/kJ
Ru	: Universal gas constant of ideal gas, 8.314472 kJ/K/kmole
MW	: Mole weight of ideal gas

UNITS

Length, diameter	: m	Absolute temperature	: oK
Mass flow rate	: kg/s	Enthalpy	: kJ/kg

Area : m2
Absolute pressure : bar a
Velocity, Sonic Velocity : m/s
Specific volume : m3/kg

TEXT

1. FANNO FLOW

Fanno flow is compressible flow that ideal gas flows through a duct having constant cross section area with friction in adiabatic process with the following assumption.

1) Steady one-dimensional flow
2) Constant friction factor over length of duct
3) Adiabatic flow(no heat transfer through wall)
4) Effective conduit diameter is four(4) times hydraulic radius (cross-sectioned area divided by wetted perimeter)
5) Elevation changes are unimportant compared with friction effects.
6) No work added to or extracted from the flow

2. CHOKE STATE

In Fanno flow, when back pressure of duct is lowered, then velocity and mass flow rate increase. At the same time, specific volume also increases.

Down to a certain back pressure, mass flow rate increases because increasing rate of velocity is higher than increasing rate of specific volume. However, below the certain back pressure, mass flow rate does not increase rather decrease, because increasing rate of specific volume becomes higher than increasing rate of velocity.

That means, a state exists that mass flow rate reaches a maximum value, and it is called as Choke State. If the back pressure is lowered further below the choke state, excess energy dissipates by turbulence and shock wave in case of duct.
The velocity at exit cross section of duct reaches sonic velocity when flow becomes choke state.

3. FANNO FLOW EQUATIONS

Fanno flow equations are as below and expressed by friction resistance coefficient(K(fLmax/D)) and Mach number at inlet cross section of duct(M1), with the condition that Mach number at exit cross section of duct is 1.

K(fLmax/D) = 1 / k x (1 / M1^2 – 1) + (k + 1) / 2 / k x ln(M1^2 x (k + 1) / ((k – 1) x M1^2 + 2)) -- (Eq. 1)

T2c / T1 = ((k – 1) x M1^2 + 2) / (k + 1) -- (Eq. 2)

P2c / P1 = M1 x (((k – 1) x M1^2 + 2) / (k + 1))^(0.5) --------------------------------------- (Eq. 3)

Note : The equations above have a assumption that specific heat ratio of ideal gas(k) is constant.

4. INDUSTRIAL FANNO FLOW ANALYSIS

While the Fanno flow analysis used in school includes both subsonic and supersonic states, the analysis used in the industry deals only subsonic including choke state.
Supersonic Fanno flow exists when Mach number at inlet cross section of duct is supersonic that can be realized by installing a supersonic nozzle in front of Fanno duct. However, in the industry, such case does not exist.

The Fanno table used in school contains M1, P2c/P1, T2c/T1 and K(fLmax/D) as below, which has difficulty to be used in the analysis of industrial purpose.
In industrial analysis, source ideal gas state(P0, T0) is given rather than M1. Inversely, M1 should be calculated instead of being given.

Scientific Fanno Table for Ideal Gas (Specific Heat Ratio = 1.3)

M1	T1/T2c	P1/P2c	v1/v2c	K(fLmax/D)
0.00	1.1500	∞	0.000	∞
0.10	1.1483	10.716	0.107	72.202
0.50	1.1084	2.106	0.526	1.172
1.00	1.0000	1.000	1.000	0.000
2.00	0.7188	0.424	1.696	0.357
5.00	0.2421	0.098	2.460	0.854
∞	0.0000	0.000	2.769	1.033

5. FREE FLOW AND GIVEN FLOW

There are two kinds of industrial Fanno flow analysis, that are Free Flow and Given Flow

Free Flow is the case that there is no flow control device in front of Fanno duct, in which the maximum flow passes through.
Meanwhile, Given Flow is the case that there is a flow control device in front of Fanno duct, in which the flow to pass through is controlled, i.e. given.

The Given Flow should be equal to or less than Free Flow, because Free Flow is the maximum flow that the duct passes through

6. INDUSTRIAL FANNO FLOW CALCULATION

6.1 FREE FLOW

For Free Flow calculation, source ideal gas state(P0, T0) and friction resistance coefficient of duct(K(fLmax/D)) are given, and then calculated are the maximum mass flow rate per duct cross section area(W/Amax) and ideal gas states at the inlet and exit cross sections of duct(P1, T1, P2c and T2c)
In case of Free Flow, the process between source state and the inlet cross section of duct is isentropic process.

Industrial Free Flow calculation is done by the following steps.

1) Find M1 that results in the given K(fLmax/D) in Eq. 1 by using try-and-error method.
2) Find T1 that results in the M1 found in step 1) by sequential calculation of the following ideal gas equations by using try-and-error method.

   $H0 = Ru / MW \times k / (k - 1) \times T0$ -- (Eq. 4)
   $H1 = Ru / MW \times k / (k - 1) \times T1$ -- (Eq. 5)
   $Vel1 = ((H0 - H1) \times 2 \times g \times J)^{(0.5)}$ --- (Eq. 6)
   $SonicVel1 = (k \times Ru / MW \times T1 / 10)^{(0.5)} \times 100$ -- (Eq. 7)
   $M1 = Vel1 / SonicVel1$ --- (Eq. 8)
3) Calculate P1 by isentropic process equation.
   $P1 = (T1 / T0)^{(k / (k - 1))} \times P0$ --- (Eq. 9)

4) Calculate v1 and W/Amax by ideal gas equation and continuity equation.
v1 = Ru / MW x T1 / P1 / 100 --- (Eq. 10)
W/Amax = Vel1 / v1 --- (Eq. 11)

5) Calculate P2c and T2c by Eq. 2 and Eq. 3.

Several calculation examples for Free Flow are presented below.

Given :	Ex. 1	Ex. 2	Ex. 3	Ex. 4
k	1.3	1.3	1.4	1.4
Mole weight (MW)	18	28	18	28
K(fLmax/D)	10	10	10	10
P0, bar a	150	150	150	150
T0, oK	500	500	500	500

Calculation :

M1	0.24172	0.24172	0.23388	0.23388
(Eq. 1) K(fLmax/D)	10	10	10	10
T1	495.656	495.656	494.589	494.589
(Eq. 4) H0, kJ/kg	1000.82	643.38	808.35	519.65
(Eq. 5) H1, kJ/kg	992.12	637.79	799.60	514.03
(Eq. 6) Vel1, m/s	131.87	105.73	132.27	106.05
(Eq. 7) SonicVel1, m/s	545.56	437.42	565.55	453.45
(Eq. 8) M1	0.24172	0.24172	0.23388	0.23388
(Eq. 9) P1, bar a	144.434	144.434	144.395	144.395
(Eq. 10) v1, m3/kg	0.015852	0.010190	0.015822	0.010171
(Eq. 11) W/Amax, kg/s/m2	8319.13	10375.78	8360.05	10426.82
(Eq. 2) T2c, oK	434.783	434.783	416.667	416.667
(Eq. 3) P2c, bar a	32.698	32.698	30.997	30.997

6.2 GIVEN FLOW

For Given Flow calculation, mass flow rate per duct cross section area(W/A) as well as source ideal gas state(P0, T0) and friction resistance coefficient of duct(K(fLmax/D)) are given, and then calculated are ideal gas states at the inlet and exit cross sections of duct(P1, T1, P2c and T2c)

Calculation steps of Given Flow are same with those of Free Flow except step 3) and 4) as below.

3) Calculate v1 by continuity equation using the given W/A.
   v1 = Vel1 / (W/A) -- (Eq. 12)

4) Calculate P1 by ideal gas equation.
   P1 = Ru / MW x T1 / v1 / 100 --- (Eq. 13)

In case of Given Flow, the process between source state and the inlet cross section of duct is polytropic process.

Several calculation examples for Given Flow are presented below.

Given :	Ex. 5	Ex. 6	Ex. 7	Ex. 8
k	1.3	1.3	1.4	1.4
Mole weight (MW)	18	28	18	28
K(fLmax/D)	10	10	10	10
P0, bar a	150	150	150	150
T0, oK	500	500	500	500
W/A, kg/s/m2	3000	3000	3000	3000

Calculation :				
M1	0.24172	0.24172	0.23388	0.23388
(Eq. 1) K(fLmax/D)	10	10	10	10
T1	495.656	495.656	494.589	494.589
(Eq. 4) H0, kJ/kg	1000.82	643.38	808.35	519.65
(Eq. 5) H1, kJ/kg	992.12	637.79	799.60	514.03
(Eq. 6) Vel1, m/s	131.87	105.73	132.27	106.05
(Eq. 7) SonicVel1, m/s	545.56	437.42	565.55	453.45
(Eq. 8) M1	0.24172	0.24172	0.23388	0.23388
(Eq. 12) v1, m3/kg	0.043957	0.035244	0.044090	0.035351
(Eq. 13) P1, bar a	52.085	41.761	51.816	41.545
(Eq. 2) T2c, oK	434.783	434.783	416.667	416.667
(Eq. 3) P2c, bar a	11.791	9.454	11.123	8.918

7. INDUSTRIAL FANNO FLOW CALCULATION BY TABLE

7.1 FREE FLOW

When a duct with K(fLmax/D) of zero(0) is in choke state, the states of inlet and exit cross section of the duct are same as the choke state of a nozzle at throat that has same source ideal gas state.

When preparing a table that contains ratio of W/Amax, P1, T1, P2c and T2c of Fanno duct to those of the choke state of a nozzle per various K(fLmax/D) values of duct, it is found that the table values are identical regardless of source ideal gas state.

On the other hand, when source ideal gas state and gas mole weight are different from another, it is found that certain relations between the two exist in the values of W/Amax, P1, T1, P2c and T2c values of Fanno duct and nozzle as below.

1) W/Amax value of Fanno duct against nozzle is linearly proportional to the ratio of absolute pressure of source ideal gas(P0), inversely proportional to square root of the ratio of absolute temperature of source ideal gas(T0), and linearly proportional to square root of the ratio of ideal gas mole weight(MW).
2) P2c value of Fanno duct against nozzle is linearly proportional to the ratio of absolute pressure of source ideal gas(P0).
3) T2c value of Fanno duct against nozzle if linearly proportional to the ratio of absolute temperature of source ideal gas(T0).

It is interesting that results of industrial Fanno flow analysis for Free Flow are easily obtained by simple equations, using the ratio table of Fanno duct to nozzle and the relations of different source ideal gas state explained above. It is no need to do iterative calculation by try-and-error method described in Clause 6.1.

Below is a part of the ratio table of Fanno duct to nozzle. The ratio table values are identical regardless of mole weight of ideal gas as well as source ideal gas state. The ratio table values vary when specific heat(k) ratio of ideal gas varies.

Ratio Table of Fanno Duct to Nozzle for Free Flow (k = 1.3)

K(fLmax/D)	dn_W/Amax, dn_P2c	dn_P1	dn_T1
Nozzle	1.00000	1.00000	1.00000

0.0001	0.99994	1.00971	1.00223
1	0.76389	1.54148	1.10502
10	0.39944	1.76442	1.14001
50	0.20152	1.81567	1.14757

Ratio Table of Fanno Duct to Nozzle for Free Flow (k = 1.4)

K(fLmax/D)	dn_W/Amax, dn_P2c	dn_P1	dn_T1
Nozzle	1.00000	1.00000	1.00000
0.0001	0.99993	1.01062	1.00302
1	0.75559	1.58643	1.14094
10	0.39117	1.82220	1.18701
50	0.19668	1.87559	1.19685

Note : dn_W/Amax = W/Amax / W/A_nozzle
dn_P2c = P2c / Pc_nozzle
dn_P1 = P1 / Pc_nozzle
dn_T1 = T1 / Tc_nozzle
dn_T2c = 1, T2c value is identical to Tc_nozzle regardless of K(fLmax/D).

Below are the interesting equations that calculate results of Free Flow analysis simply by table.

W/Amax = dn_W/Amax x W/A_nozzle x P0 / P0_nozzle / (T0 / T0_nozzle)^(0.5) * (MW / MW_nozzle)^(0.5) -- (Eq. 14)

P1 = dn_P1 x Pc_nozzle x P0 / P0_nozzle --- (Eq. 15)

T1 = dn_T1 x Tc_nozzle x T0 / T0_nozzle --- (Eq. 16)

P2c = dn_P2c x Pc_nozzle x P0 / P0_nozzle --------------------------------------- (Eq. 17)

T2c = Tc_nozzle x T0 / T0_nozzle --- (Eq. 18)

A set of nozzle choke state values is required for use of the above equations. It is noted that whatever nozzle choke state is used, calculation results are same.

Below is a set of nozzle choke state with source ideal gas state) of 100 bar a pressure and 1000 oK temperature.

For k = 1.3 :

MW_nozzle	P0_nozzle	T0_nozzle	W/A_nozzle	Pc_nozzle	Tc_nozzle
18	100	1000	9817.830	54.5728	869.5652
	bar a	oK	Kg/s/m2	bar a	oK

For k = 1.4 :

MW_nozzle	P0_nozzle	T0_nozzle	W/A_nozzle	Pc_nozzle	Tc_nozzle
18	100	1000	10074.863	52.8282	833.3333
	bar a	oK	Kg/s/m2	bar a	oK

The calculation examples for Free Flow presented in Clause 6.1 are calculated by table as below.

Given :	Ex. 1	Ex. 2	Ex. 3	Ex. 4
k	1.3	1.3	1.4	1.4
Mole weight (MW)	18	28	18	28
K(fLmax/D)	10	10	10	10
P0, bar a	150	150	150	150
T0, oK	500	500	500	500

Calculation :				
(Eq. 14) W/Amax, kg/s/m2	8319.04	10375.67	8360.09	10426.86
(Eq. 15) P1, bar a	144.434	144.434	144.395	144.395
(Eq. 16) T1, oK	495.657	495.657	494.587	494.587
(Eq. 17) P2c, bar a	32.698	32.698	30.997	30.997
(Eq. 18) T2c, oK	434.783	434.783	416.667	416.667

The calculation results are same with those of Clause 6.1 below. Difference of result values, if exists, is matter of precision by significant figures or interpolation used in each calculation, not matter of accuracy of the method or equations.

Calculation Results of Clause 6.1 :				
(Eq. 11) W/Amax, kg/s/m2	8319.13	10375.78	8360.05	10426.82
(Eq. 9) P1, bar a	144.434	144.434	144.395	144.395
T1	495.656	495.656	494.589	494.589
(Eq. 3) P2c, bar a	32.698	32.698	30.997	30.997
(Eq. 2) T2c, oK	434.783	434.783	416.667	416.667

7.2 GIVEN FLOW

As in Free Flow, it is found interestingly that certain relations among various different conditions exist as below.

1) T2c of Fanno duct is identical regardless of K(fLmax/D) of Fanno duct, the given W/A and source ideal gas absolute pressure(P0), but linearly proportional to the ratio of absolute temperature of source ideal gas(T0).
2) P2c of Fanno duct is identical regardless of K(fLmax/D) of Fanno duct, if identical are the given W/A, and source ideal gas absolute pressure(P0) and source ideal gas absolute temperature(T0).
   On the other hand, P2c change rate against W/A is linearly constant, and the rate is identical regardless of source ideal gas absolute pressure(P0) but linearly proportional to square root of change rate of source ideal gas absolute temperature(T0).
   And, P2c is linearly proportional to square root of the ratio of ideal gas mole weight(MW).
3) Ratios of P1 to P2c and T1 to T2c are identical regardless of the given W/A, source ideal gas absolute pressure(P0) and source ideal gas absolute temperature(T0), but different depending on K(fLmax/D) only.

Like in Free Flow, the results of industrial Fanno flow analysis for Given Flow are easily obtained by simple equations, using the ratio table of P1/P2c and T1/T2c described in 3) and the relations described in 1) and 2) above. It is no need to do iterative calculation by try-and-error method described in Clause 6.2.

Below is a part of the ratio table of P1/P2c and T1/T2c against K(fLmax/D). The ratio table values are identical regardless of the given flow W/A, mole weight of ideal gas as well as source ideal gas state. The ratio table values vary when specific heat(k) ratio of ideal gas varies.

Ratio Table of P1/P2c and T1/T2c for Given Flow (k = 1.3)

K(fLmax/D)	P1/P2c	T1/T2c
0.0001	1.00978	1.00223
1	2.01795	1.10502
10	4.41719	1.14001
50	9.00990	1.14757

Ratio Table of P1/P2c and T1/T2c for Given Flow (k = 1.4)

K(fLmax/D)	P1/P2c	T1/T2c
0.0001	1.01069	1.00302
1	2.09959	1.14094
10	4.65835	1.18701
50	9.53613	1.19685

Below are the equations that calculate results of Givne Flow analysis simply by table.

P2c = ((W/A – W/A_ref) x P2c/(W/A)_ref + P2c_ref) x (T0 / T0_ref)^(0.5) / (MW / MW_ref)^(0.5) -- (Eq. 19)

T2c = T2c_ref x T0 / T0_ref --- (Eq. 20)

P1 = P1/P2c_table x P2c -- (Eq. 21)

T1 = T1/T2c_table x T2c -- (Eq. 22)

A set of reference choke state values of a Given Flow is required for use of the equations above. It is noted that whatever reference choke state is used, calculation results are same.

Below is a set of reference choke state values of a Given Flow with source ideal gas state of 100 bar a pressure and 1000 oK temperature and a Given Flow of 1000 kg/s/m2.

For k = 1.3 :

MW_ref	P0_ref	T0_ref	W/A_ref	P2c_ref	P2c/(W/A)_ref	T2c_ref
18	100	1000	1000	5.55854	5.55854E-03	869.565
	bar a	oK	Kg/s/m2	bar a	bar/(kg/s/m2)	oK

For k = 1.4 :

MW_ref	P0_ref	T0_ref	W/A_ref	P2c_ref	P2c/(W/A)_ref	T2c_ref
18	100	1000	1000	5.24356	5.24356E-03	833.333
	bar a	oK	Kg/s/m2	bar a	bar/(kg/s/m2)	oK

Note : 1. The reference choke state values are identical, whatever K(fLmax/D) value is used.
2. The given flow value of the reference choke state should be equal to or less than the free flow value at the same condition.

The calculation examples for Given Flow presented in Clause 6.2 are calculated by table as below.

Given :	Ex. 5	Ex. 6	Ex. 7	Ex. 8
k	1.3	1.3	1.4	1.4
Mole weight (MW)	18	28	18	28
K(fLmax/D)	10	10	10	10
P0, bar a	150	150	150	150
T0, oK	500	500	500	500
W/A, kg/s/m2	3000	3000	3000	3000

Calculation :				
(Eq. 19) P2c, bar a	11.791	9.454	11.123	8.918
(Eq. 20) T2c, oK	434.783	434.783	416.667	416.667
(Eq. 21) P1, bar a	52.085	41.761	51.816	41.545
(Eq. 22) T1, oK	495.657	495.657	494.587	494.587

The calculation results are same with those of Clause 6.2 below. Difference of result values, if exists, is matter of precision by significant figures or interpolation used in each calculation, not matter of accuracy of the method or equations.

Calculation Results of Clause 6.2 :				
(Eq. 3) P2c, bar a	11.791	9.454	11.123	8.918
(Eq. 2) T2c, oK	434.783	434.783	416.667	416.667
(Eq. 13) P1, bar a	52.085	41.761	51.816	41.545
T1	495.656	495.656	494.589	494.589

ATTACHMENTS

Attachment # 1 : Ratio Table of Fanno Duct to Nozzle for Free Flow (Cp/Cv = 1.3)
Attachment #2 : Ratio Table of Fanno Duct to Nozzle for Free Flow (Cp/Cv = 1.4)
Attachment #3 : Ratio Table of P1/P2c and T1/T2c for Given Flow (Cp/Cv = 1.3)
Attachment #4 : Ratio Table of P1/P2c and T1/T2c for Given Flow (Cp/Cv = 1.4)
(End)